Karlsruhe Institute of Technology

Robotik I: Einführung in die Robotik
Übung 3: Inverse Kinematik und Dynamik

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Hochperformante Humanoide Technologien (H^2T)

Aufgabe 1: Differentielle Inverse Kinematik

- SCARA-Roboter mit
  - Einem Translationsgelenk d_1
  - Zwei Rotationsgelenken θ_2, θ_3
  - Konfiguration $\boldsymbol{q} = (d_1,\ \theta_2,\ \theta_3)$

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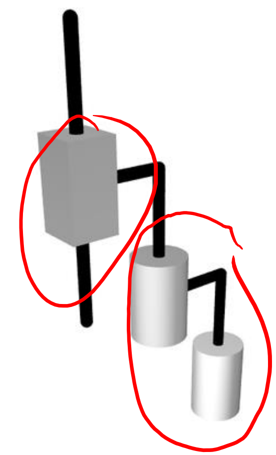

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- Vorwärtskinematik (nur Position):

$$f(\boldsymbol{q}) = \begin{pmatrix} -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2) \\ 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2) \\ d_1 \end{pmatrix}$$

End-Effektor Geschwindigkeiten

- Die Jacobi-Matrix setzt kartesische End-Effektor-Geschwindigkeiten in Relation zu Gelenkwinkelgeschwindigkeiten

$$\dot{\boldsymbol{x}}(t) = J_f\big(\theta(t)\big) \cdot \dot{\theta}(t)$$

$$J^{-1} \dot{x} = \dot{q}$$

- Die folgenden Probleme können mit dieser Beziehung gelöst werden

1. Gegeben eine kartesische End-Effektor-Geschwindigkeit, welche Gelenkwinkelgeschwindigkeiten sind notwendig, um diese zu realisieren?
2. Gegeben die Gelenkwinkelgeschwindigkeiten, welche kartesische End-Effektor-Geschwindigkeit wird damit realisiert?

Aufgabe 1: Inverse Kinematik

- Vorwärtskinematik (nur Position):

$$f(\boldsymbol{q}) = \begin{pmatrix} -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2) \\ 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2) \\ d_1 \end{pmatrix}$$

- Inverse Kinematik

$$\dot{\boldsymbol{q}} = J^{-1}(\boldsymbol{q}) \cdot \dot{\boldsymbol{x}}$$

- Teilaufgaben:
  - 1.1: Bestimmen Sie die inverse Jacobi-Matrix $J^{-1}(\boldsymbol{q})$.
  - 1.2: Bestimmen Sie $\dot{\boldsymbol{q}}$ bei gegebenem $\boldsymbol{q}$ und $\dot{\boldsymbol{x}}$.
  - 1.3: In welchen Stellungen treten Singularitäten auf?

Aufgabe 1.1: Inverse Jacobi-Matrix

$$f(\boldsymbol{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2) \\ 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2) \\ d_1 \end{pmatrix}$$

- Jacobi-Matrix:

$$J(q) = \begin{pmatrix} \frac{\partial f}{\partial d_1} & \frac{\partial f}{\partial \theta_2} & \frac{\partial f}{\partial \theta_3} \end{pmatrix}$$

$$\frac{\partial f}{\partial d_1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$f(\boldsymbol{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2) \\ 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2) \\ d_1 \end{pmatrix}$$

- Jacobi-Matrix:

$$J(\boldsymbol{q}) = \left(\frac{\partial f}{\partial d_1}, \frac{\partial f}{\partial \theta_2}, \frac{\partial f}{\partial \theta_3} \right)$$

$$\frac{\partial f}{\partial d_1} =$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$f(\boldsymbol{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2) \\ 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2) \\ d_1 \end{pmatrix}$$

- Jacobi-Matrix:

$$J(\boldsymbol{q}) = \left(\frac{\partial f}{\partial d_1}, \frac{\partial f}{\partial \theta_2}, \frac{\partial f}{\partial \theta_3} \right)$$

$$\frac{\partial f}{\partial d_1} = \begin{pmatrix} 0 \\ 0 \\ \frac{\partial}{\partial d_1}(d_1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für x vereinfachen

$$x = -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2)$$

$$= -500\left(\sin(\theta_2)\cos(\theta_3) + \cos(\theta_2)\cdot\sin(\theta_3)\right) - 500 \sin\theta_2$$

$$= -500 \sin(\theta_2 + \theta_3) - 500 \sin\theta_2$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für x vereinfachen

$$x = -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2)$$

$$x = -500 \cdot (\sin(\theta_2) \cdot \cos(\theta_3) + \cos(\theta_2) \cdot \sin(\theta_3)) - 500 \cdot \sin(\theta_2)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für x vereinfachen

$$x = -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2)$$

$$x = -500 \cdot (\sin(\theta_2) \cdot \cos(\theta_3) + \cos(\theta_2) \cdot \sin(\theta_3)) - 500 \cdot \sin(\theta_2)$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für x vereinfachen

$$x = -500 \cdot \sin(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \cos(\theta_2) \cdot \sin(\theta_3) - 500 \cdot \sin(\theta_2)$$

$$x = -500 \cdot (\sin(\theta_2) \cdot \cos(\theta_3) + \cos(\theta_2) \cdot \sin(\theta_3)) - 500 \cdot \sin(\theta_2)$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$x = -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für y vereinfachen

$$y = 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$= 500 \left(\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3 \right) + \ldots$

$\cos(\alpha + \beta)$

$= \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$= 500 \cdot \cos(\theta_2 + \theta_3) + 100 + 500 \cdot \cos\theta_2$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für y vereinfachen

$$y = 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$$y = 500 \cdot (\cos(\theta_2) \cdot \cos(\theta_3) - \sin(\theta_2) \cdot \sin(\theta_3)) + 100 + 500 \cdot \cos(\theta_2)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Ausdruck für y vereinfachen

$$y = 500 \cdot \cos(\theta_2) \cdot \cos(\theta_3) - 500 \cdot \sin(\theta_2) \cdot \sin(\theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$$y = 500 \cdot (\cos(\theta_2) \cdot \cos(\theta_3) - \sin(\theta_2) \cdot \sin(\theta_3)) + 100 + 500 \cdot \cos(\theta_2)$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$y = 500 \cdot \cos(\theta_2 + \theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

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$$x = -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)$$

$$\frac{\partial x}{\partial \theta_2} = -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cos(\theta_2)$$

$$y = 500 \cdot \cos(\theta_2 + \theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$$\frac{\partial y}{\partial \theta_2} = 500(-\sin(\theta_2 + \theta_3)) + 500(-\sin \theta_2)$$

$$z = d_1 \qquad \frac{\partial z}{\partial \theta_2} = 0$$

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Aufgabe 1.1: Inverse Jacobi-Matrix

$$x = -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)$$

$$\frac{\partial x}{\partial \theta_2} = -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)$$

$$y = 500 \cdot \cos(\theta_2 + \theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$$\frac{\partial y}{\partial \theta_2} = -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)$$

$$z = d_1 \qquad \frac{\partial z}{\partial \theta_2} = 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$x = -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)$$

$$\frac{\partial x}{\partial \theta_3} = -500 \cdot \cos(\theta_2 + \theta_3) \quad -$$

$$y = 500 \cdot \cos(\theta_2 + \theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$$\frac{\partial y}{\partial \theta_3} = 500 \cdot (-\sin(\theta_2 + \theta_3))$$

$$z = d_1 \qquad \frac{\partial z}{\partial \theta_3} = 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$x = -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)$$

$$\frac{\partial x}{\partial \theta_3} = -500 \cdot \cos(\theta_2 + \theta_3)$$

$$y = 500 \cdot \cos(\theta_2 + \theta_3) + 100 + 500 \cdot \cos(\theta_2)$$

$$\frac{\partial y}{\partial \theta_3} = -500 \cdot \sin(\theta_2 + \theta_3)$$

$$z = d_1 \qquad \frac{\partial z}{\partial \theta_3} = 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

- Jacobi-Matrix:

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$J \in \mathbb{R}^{3 \times 3}$

- Gesucht:

$$J^{-1}(\boldsymbol{q}) = ?$$

Aufgabe 1.1: Inverse Jacobi-Matrix

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- Matrix:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

- Invertieren:

$$\mathrm{A}^{-1} = \frac{1}{\det A} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

- Determinante (Regel von Sarrus):

$$\det A = aei + bfg + cdh - ceg - bdi - afh$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= 0 +$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500\cdot\cos(\theta_2+\theta_3) - 500\cdot\cos(\theta_2) & -500\cdot\cos(\theta_2+\theta_3) \\ 0 & -500\cdot\sin(\theta_2+\theta_3) - 500\cdot\sin(\theta_2) & -500\cdot\sin(\theta_2+\theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= 0 + (-500\cdot\cos(\theta_2+\theta_3) - 500\cdot\cos(\theta_2))\cdot(-500\cdot\sin(\theta_2+\theta_3))$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= 0 + (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) + 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

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$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= 0 + (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) + 0$$

$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

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Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= 0 + (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) + 0$$

$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)) - 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= 0 + (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) + 0$$

$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)) - 0 - 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3))$$
$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3))$$
$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= (-500) \cdot (-500) \cdot \begin{pmatrix} (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot \sin(\theta_2 + \theta_3) \\ -(\sin(\theta_2 + \theta_3) + \sin(\theta_2)) \cdot \cos(\theta_2 + \theta_3) \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3))$$
$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= (-500) \cdot (-500) \cdot \begin{pmatrix} (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot \sin(\theta_2 + \theta_3) \\ -(\sin(\theta_2 + \theta_3) + \sin(\theta_2)) \cdot \cos(\theta_2 + \theta_3) \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = aei + bfg + cdh - ceg - bdi - afh$$

$$= (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3))$$
$$-(-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= (-500) \cdot (-500) \cdot \begin{pmatrix} (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot \sin(\theta_2 + \theta_3) \\ -(\sin(\theta_2 + \theta_3) + \sin(\theta_2)) \cdot \cos(\theta_2 + \theta_3) \end{pmatrix}$$

$$= 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \begin{pmatrix} \cos(\theta_2) \cdot (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) \\ - \sin(\theta_2) \cdot (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$
$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \begin{pmatrix} \cos(\theta_2) \cdot (\sin \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_3) \\ - \sin(\theta_2) \cdot (\cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3) \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \begin{pmatrix} \cos(\theta_2) \cdot (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) \\ - \sin(\theta_2) \cdot (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) \end{pmatrix}$$

$$= 500^2 \cdot (\cos^2\theta_2 \sin\theta_3 + \sin^2\theta_2 \sin\theta_3)$$

$$= 500^2 \sin\theta_3 \underbrace{(\cos^2\theta_2 + \sin^2\theta_2)}_{1}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \begin{pmatrix} \cos(\theta_2) \cdot (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) \\ - \sin(\theta_2) \cdot (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) \end{pmatrix}$$

$$= 500^2 \cdot (\cos^2\theta_2 \sin\theta_3 + \sin^2\theta_2 \sin\theta_3)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \begin{pmatrix} \cos(\theta_2) \cdot (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) \\ - \sin(\theta_2) \cdot (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) \end{pmatrix}$$

$$= 500^2 \cdot (\cos^2\theta_2 \sin\theta_3 + \sin^2\theta_2 \sin\theta_3)$$

$$= 500^2 \cdot \sin\theta_3 (\cos^2\theta_2 + \sin^2\theta_2)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \sin(\theta_2) \cdot \cos(\theta_2 + \theta_3))$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \begin{pmatrix} \cos(\theta_2) \cdot (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) \\ - \sin(\theta_2) \cdot (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) \end{pmatrix}$$

$$= 500^2 \cdot (\cos^2\theta_2 \sin\theta_3 + \sin^2\theta_2 \sin\theta_3)$$

$$= 500^2 \cdot \sin\theta_3 (\cos^2\theta_2 + \sin^2\theta_2)$$
$$= \boxed{500^2 \cdot \sin\theta_3 \cdot 1}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \sin\theta_3$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{\det J(\boldsymbol{q})} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \sin\theta_3$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{\det J(\boldsymbol{q})} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

$$ei - fh = 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

$$ei - fh = 0$$
$$ch - bi = 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

$ei - fh = 0$

$ch - bi = 0$

$cd - af = 0$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

$ei - fh = 0$

$ch - bi = 0$

$cd - af = 0$

$ae - bd = 0$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$bf - ce =$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$bf - ce = -500 \cdot (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) - (-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$bf - ce = -500 \cdot (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) - (-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= 500^2 \cdot (\cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) + \cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \cos(\theta_2 +$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$bf - ce = -500 \cdot (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) - (-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= 500^2 \cdot \begin{pmatrix} \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) + \cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) \\ - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2) \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & bf - ce \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$bf - ce = -500 \cdot (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) - (-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= 500^2 \cdot \begin{pmatrix} \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) + \cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) \\ - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2) \end{pmatrix}$$

$$= 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2)) = \det J(q)$$

$$= 500^2 \cdot \sin\theta_3$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$bf - ce = -500 \cdot (\cos(\theta_2 + \theta_3) + \cos(\theta_2)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3)) - (-500 \cdot \cos(\theta_2 + \theta_3)) \cdot (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2))$$

$$= 500^2 \cdot \begin{pmatrix} \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) + \cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) \\ - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2) \end{pmatrix}$$

$$= 500^2 \cdot (\cos(\theta_2) \cdot \sin(\theta_2 + \theta_3) - \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2)) = 500^2 \cdot \sin\theta_3$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$fg - di =$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ fg - di & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$fg - di = -500 \cdot \sin(\theta_2 + \theta_3) - 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$ai - cg =$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500\cdot\cos(\theta_2+\theta_3) - 500\cdot\cos(\theta_2) & -500\cdot\cos(\theta_2+\theta_3) \\ 0 & -500\cdot\sin(\theta_2+\theta_3) - 500\cdot\sin(\theta_2) & -500\cdot\sin(\theta_2+\theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2\cdot\sin\theta_3}\begin{pmatrix} 0 & 0 & 500^2\cdot\sin\theta_3 \\ -500\cdot\sin(\theta_2+\theta_3) & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$ai - cg = 0 + (-500\cdot\cos(\theta_2+\theta_3)\cdot 1)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin \theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin \theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & ai - cg & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$ai - cg = 0 - (-500 \cdot \cos(\theta_2 + \theta_3) \cdot 1) = 500 \cdot \cos(\theta_2 + \theta_3)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$dh - eg =$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$dh - eg = 0 \cdot 0 + (+500 \cdot \sin(\theta_2 + \theta_3) + 500 \cdot \sin(\theta_2)) \cdot 1$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ dh - eg & bg - ah & 0 \end{pmatrix}$$

$$dh - eg = 0 \cdot 0 - (-500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2)) \cdot 1$$

$$= 500 \cdot \sin(\theta_2 + \theta_3) + 500 \cdot \sin(\theta_2)$$

$$= 500 \cdot (\sin(\theta_2 + \theta_3) + \sin(\theta_2))$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ 500 \cdot (\sin(\theta_2 + \theta_3) + \sin(\theta_2)) & bg - ah & 0 \end{pmatrix}$$

$bg - ah =$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ 500 \cdot (\sin(\theta_2 + \theta_3) + \sin(\theta_2)) & bg - ah & 0 \end{pmatrix}$$

$$bg - ah = (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot 1 - 0 \cdot 0$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ 500 \cdot (\sin(\theta_2 + \theta_3) + \sin(\theta_2)) & bg - ah & 0 \end{pmatrix}$$

$$bg - ah = (-500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)) \cdot 1 - 0 \cdot 0$$

$$= -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2)$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3} \begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2 + \theta_3) & 500 \cdot \cos(\theta_2 + \theta_3) & 0 \\ 500 \cdot (\sin(\theta_2 + \theta_3) + \sin(\theta_2)) & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & 0 \end{pmatrix}$$

Aufgabe 1.1: Inverse Jacobi-Matrix

$$J(\boldsymbol{q})^{-1} = \frac{1}{500^2 \cdot \sin\theta_3}\begin{pmatrix} 0 & 0 & 500^2 \cdot \sin\theta_3 \\ -500 \cdot \sin(\theta_2+\theta_3) & 500 \cdot \cos(\theta_2+\theta_3) & 0 \\ 500 \cdot (\sin(\theta_2+\theta_3)+\sin(\theta_2)) & -500 \cdot \cos(\theta_2+\theta_3) - 500 \cdot \cos(\theta_2) & 0 \end{pmatrix}$$

$$J(\boldsymbol{q})^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{\sin(\theta_2+\theta_3)}{500 \cdot \sin\theta_3} & \frac{\cos(\theta_2+\theta_3)}{500 \cdot \sin\theta_3} & 0 \\ \frac{\sin(\theta_2+\theta_3)+\sin(\theta_2)}{500 \cdot \sin\theta_3} & \frac{-\cos(\theta_2+\theta_3)-\cos(\theta_2)}{500 \cdot \sin\theta_3} & 0 \end{pmatrix}$$

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$J(\boldsymbol{q})^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{\sin(\theta_2+\theta_3)}{500 \cdot \sin\theta_3} & \frac{\cos(\theta_2+\theta_3)}{500 \cdot \sin\theta_3} & 0 \\ \frac{\sin(\theta_2+\theta_3)+\sin(\theta_2)}{500 \cdot \sin\theta_3} & \frac{-\cos(\theta_2+\theta_3)-\cos(\theta_2)}{500 \cdot \sin\theta_3} & 0 \end{pmatrix}$$

- Gegeben:
  - Zustand des Roboters $\boldsymbol{q} = (d_1, \theta_2, \theta_3)^T = \left(1, 0, \frac{\pi}{2}\right)^T$
  - EEF-Geschwindigkeit $\dot{\boldsymbol{p}} = (1000, 0, 0)^T$
- Gesucht:
  - Gelenkwinkelgeschwindigkeit $\dot{\boldsymbol{q}}$, die die EEF-Geschwindigkeit $\dot{\boldsymbol{p}}$ erzeugt

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$J(\boldsymbol{q})^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -\dfrac{\sin(\theta_2 + \theta_3)}{500 \cdot \sin\theta_3} & \dfrac{\cos(\theta_2 + \theta_3)}{500 \cdot \sin\theta_3} & 0 \\ \dfrac{\sin(\theta_2 + \theta_3) + \sin(\theta_2)}{500 \cdot \sin\theta_3} & \dfrac{-\cos(\theta_2 + \theta_3) - \cos(\theta_2)}{500 \cdot \sin\theta_3} & 0 \end{pmatrix}$$

$$J\left(\begin{pmatrix} 1 \\ 0 \\ \frac{\pi}{2} \end{pmatrix}\right)^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -\dfrac{\sin(\theta_2 + \theta_3)}{500 \cdot \sin\theta_3} & \dfrac{\cos(\theta_2 + \theta_3)}{500 \cdot \sin\theta_3} & 0 \\ \dfrac{\sin(\theta_2 + \theta_3) + \sin(\theta_2)}{500 \cdot \sin\theta_3} & \dfrac{-\cos(\theta_2 + \theta_3) - \cos(\theta_2)}{500 \cdot \sin\theta_3} & 0 \end{pmatrix}$$

q J_1

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$J\left(\begin{pmatrix}1\\0\\\frac{\pi}{2}\end{pmatrix}\right)^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -\dfrac{\sin\left(0+\frac{\pi}{2}\right)}{500\cdot\sin\frac{\pi}{2}} & \dfrac{\cos\left(0+\frac{\pi}{2}\right)}{500\cdot\sin\frac{\pi}{2}} & 0 \\ \dfrac{\sin\left(0+\frac{\pi}{2}\right)+\sin(0)}{500\cdot\sin\frac{\pi}{2}} & \dfrac{-\cos\left(0+\frac{\pi}{2}\right)-\cos(0)}{500\cdot\sin\frac{\pi}{2}} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{500} & 0 & 0 \\ \frac{1}{500} & -\frac{1}{500} & 0 \end{pmatrix}$$

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$J\left(\begin{pmatrix}1\\0\\\frac{\pi}{2}\end{pmatrix}\right)^{-1}=\begin{pmatrix}0 & 0 & 1\\ -\frac{\sin\left(0+\frac{\pi}{2}\right)}{500\cdot\sin\frac{\pi}{2}} & \frac{\cos\left(0+\frac{\pi}{2}\right)}{500\cdot\sin\frac{\pi}{2}} & 0\\ \frac{\sin\left(0+\frac{\pi}{2}\right)+\sin(0)}{500\cdot\sin\frac{\pi}{2}} & \frac{-\cos\left(0+\frac{\pi}{2}\right)-\cos(0)}{500\cdot\sin\frac{\pi}{2}} & 0\end{pmatrix}$$

$$J\left(\begin{pmatrix}1\\0\\\frac{\pi}{2}\end{pmatrix}\right)^{-1}=\begin{pmatrix}0 & 0 & 1\\ -\frac{1}{500\cdot 1} & \frac{0}{500\cdot 1} & 0\\ \frac{1+0}{500\cdot 1} & \frac{-0-1}{500\cdot 1} & 0\end{pmatrix}$$

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$J\left(\begin{pmatrix}1\\0\\\frac{\pi}{2}\end{pmatrix}\right)^{-1}=\begin{pmatrix}0 & 0 & 1\\ -\frac{\sin\left(0+\frac{\pi}{2}\right)}{500\cdot\sin\frac{\pi}{2}} & \frac{\cos\left(0+\frac{\pi}{2}\right)}{500\cdot\sin\frac{\pi}{2}} & 0\\ \frac{\sin\left(0+\frac{\pi}{2}\right)+\sin(0)}{500\cdot\sin\frac{\pi}{2}} & \frac{-\cos\left(0+\frac{\pi}{2}\right)-\cos(0)}{500\cdot\sin\frac{\pi}{2}} & 0\end{pmatrix}$$

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Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$\dot{\boldsymbol{q}} = J(\boldsymbol{q})^{-1} \cdot \dot{\boldsymbol{p}}$$

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$\dot{\boldsymbol{q}} = J(\boldsymbol{q})^{-1} \cdot \dot{\boldsymbol{p}}$$

$$\dot{\boldsymbol{q}} = J\left(\begin{pmatrix} 1 \\ 0 \\ \frac{\pi}{2} \end{pmatrix}\right)^{-1} \cdot \begin{pmatrix} 1000 \\ 0 \\ 0 \end{pmatrix}$$

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

$$\dot{\boldsymbol{q}} = J(\boldsymbol{q})^{-1} \cdot \dot{\boldsymbol{p}}$$

$$\dot{\boldsymbol{q}} = J\left(\begin{pmatrix} 1 \\ 0 \\ \frac{\pi}{2} \end{pmatrix}\right)^{-1} \cdot \begin{pmatrix} 1000 \\ 0 \\ 0 \end{pmatrix}$$

$$\dot{\boldsymbol{q}} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{500} & 0 & 0 \\ \frac{1}{500} & -\frac{1}{500} & 0 \end{pmatrix} \cdot \begin{pmatrix} 1000 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

$$\dot{d}_1 = 0 \frac{m}{s}$$

$$\dot{\theta}_2 = -2 \frac{rad}{s}$$

$$\dot{\theta}_3 = 2 \frac{rad}{s}$$

Aufgabe 1.2: Gelenkwinkelgeschwindigkeit

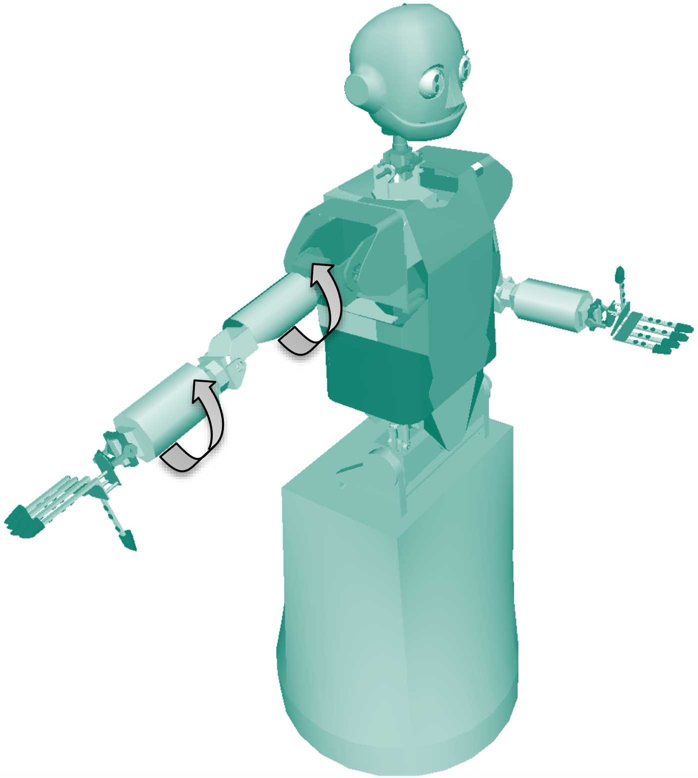
$$\dot{\boldsymbol{q}} = J(\boldsymbol{q})^{-1} \cdot \dot{\boldsymbol{p}}$$

$$\dot{\boldsymbol{q}} = J\left(\begin{pmatrix} 1 \\ 0 \\ \frac{\pi}{2} \end{pmatrix}\right)^{-1} \cdot \begin{pmatrix} 1000 \\ 0 \\ 0 \end{pmatrix}$$

$$\dot{\boldsymbol{q}} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{500} & 0 & 0 \\ \frac{1}{500} & -\frac{1}{500} & 0 \end{pmatrix} \cdot \begin{pmatrix} 1000 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

Aufgabe 1.3: Singularitäten

- Eine kinematische Kette ist in einer **singulären Konfiguration**, wenn die zugehörige Jacobi-Matrix nicht vollen Rang hat
  - Zwei oder mehr Spalten von J_f sind linear abhängig
- Die Jacobi-Matrix ist nicht invertierbar
  - Bestimmte Bewegungen unmöglich
- In der Umgebung von Singularitäten können **große Gelenkgeschwindigkeiten** nötig werden, um eine End-Effektor-Geschwindigkeit zu halten.



Aufgabe 1.3: Singularitäten

- Eine quadratische Matrix $A \in \mathbb{R}^{n \times n}$ hat genau dann vollen Rang, wenn die Determinante ungleich Null ist:

$$\operatorname{rang} A = n \Leftrightarrow \det A \neq 0$$

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500 \cdot \cos(\theta_2 + \theta_3) - 500 \cdot \cos(\theta_2) & -500 \cdot \cos(\theta_2 + \theta_3) \\ 0 & -500 \cdot \sin(\theta_2 + \theta_3) - 500 \cdot \sin(\theta_2) & -500 \cdot \sin(\theta_2 + \theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(q) = 0 = 500^2 \cdot \sin\theta_3$$

$$\theta_3 = n \cdot \pi \quad n \in [0, 1, 2, 3, \ldots$$

$$\theta_3 = 0 \lor \theta_3 = \pi \quad \theta_3 \in [0, 2\pi)$$

Aufgabe 1.3: Singularitäten

- Eine quadratische Matrix $A \in \mathbb{R}^{n\times n}$ hat genau dann vollen Rang, wenn die Determinante ungleich Null ist:

$$\operatorname{rang} A = n \Leftrightarrow \det A \neq 0$$

$$J(\boldsymbol{q}) = \begin{pmatrix} 0 & -500\cdot\cos(\theta_2+\theta_3) - 500\cdot\cos(\theta_2) & -500\cdot\cos(\theta_2+\theta_3) \\ 0 & -500\cdot\sin(\theta_2+\theta_3) - 500\cdot\sin(\theta_2) & -500\cdot\sin(\theta_2+\theta_3) \\ 1 & 0 & 0 \end{pmatrix}$$

$$\det J(\boldsymbol{q}) = 500^2 \cdot \sin\theta_3$$

- Für Singularitäten $\boldsymbol{q}_{sing}$ der quadratischen Matrix $J(\boldsymbol{q})$ gilt:

$$\det J(\boldsymbol{q}_{sing}) = 500^2 \cdot \sin\theta_3 = 0$$

Aufgabe 1.3: Singularitäten

- Für Singularitäten $\boldsymbol{q}_{sing}$ der quadratischen Matrix $J(\boldsymbol{q})$ gilt:

$$\det J(\boldsymbol{q}_{sing}) = 500^2 \cdot \sin\theta_3 = 0$$

Aufgabe 1.3: Singularitäten

- Für Singularitäten $\boldsymbol{q}_{sing}$ der quadratischen Matrix $J(\boldsymbol{q})$ gilt:

$$\det J(\boldsymbol{q}_{sing}) = 500^2 \cdot \sin\theta_3 = 0$$

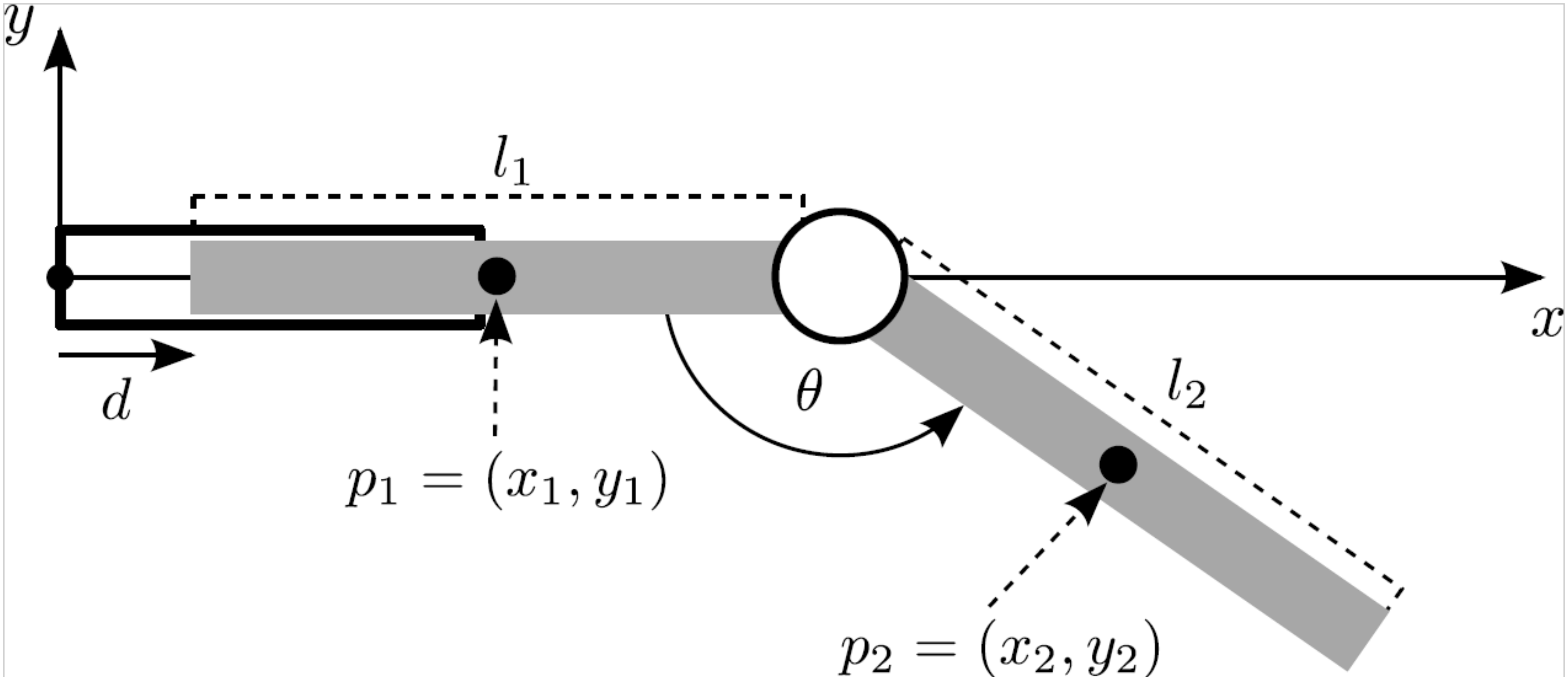
$$\sin\theta_3 = 0$$

$$\theta_3 = n \cdot \pi, n \in [0,1,2,\ldots]$$

$$\theta_3 = 0 \vee \theta_3 = \pi, \qquad \theta_3 \in [0,2\pi)$$

$$\boldsymbol{q}_{sing,1} = \begin{pmatrix} d_1 \\ \theta_2 \\ 0 \end{pmatrix}, \boldsymbol{q}_{sing,2} = \begin{pmatrix} d_1 \\ \theta_2 \\ \pi \end{pmatrix}$$

Aufgabe 2: Dynamikmodellierung nach Lagrange



- Annahmen:
  - Punktmassen in der Mitte der Segmente
  - Reibung wird vernachlässigt

- Konfiguration $\boldsymbol{q} = (d,\ \theta)^T$

Aufgabe 2: Dynamikmodellierung nach Lagrange

- Position der Punktmassen:

$$p_1 = (x_1, y_1) = (\frac{1}{2} l_1 + d, 0)$$

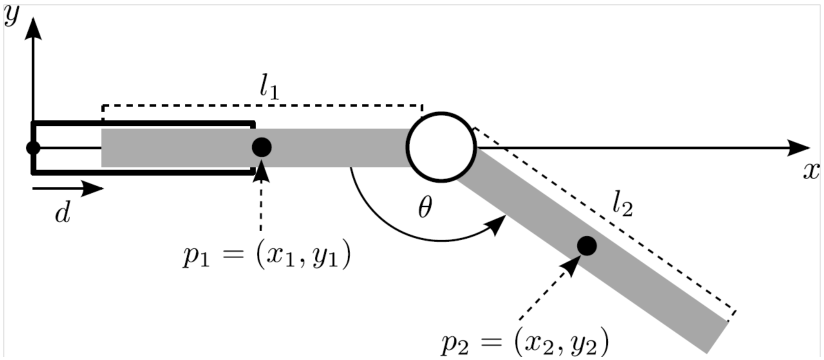
$$p_2 = (x_2, y_2) = (l_1 + d + \frac{1}{2} l_2 \cdot \cos\theta, -\frac{1}{2} l_2 \cdot \sin\theta)$$

- Position der Punktmassen:

$$p_1 = (x_1, y_1) = (\frac{1}{2} l_1 + d, 0)$$

$$p_2 = (x_2, y_2) = (l_1 + d + \frac{1}{2} l_2 \cdot \cos\theta, -\frac{1}{2} l_2 \cdot \sin\theta)$$

Aufgabe 2: Dynamikmodellierung nach Lagrange



- Konfiguration
  $\boldsymbol{q} = (d,\ \theta)^T$

- Position der Punktmassen:
  $p_1 = (x_1, y_1) = (\frac{1}{2} l_1 + d, 0)$
  $p_2 = (x_2, y_2) = (l_1 + d + \frac{1}{2} l_2 \cdot \cos\theta\,, -\frac{1}{2} l_2 \cdot \sin\theta)$

- Modellieren Sie die Dynamik des Robotersystems.
  - 2.1: Kinetische Energie für jedes Gelenk bestimmen
  - 2.2: Potentielle Energie für jedes Gelenk bestimmen
  - 2.3: Lagrange-Funktion berechnen
  - 2.4: Bewegungsgleichung aufstellen

Methode nach Lagrange (Wiederholung)

- Lagrange-Funktion:

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = E_{kin}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - E_{pot}(\boldsymbol{q})$$

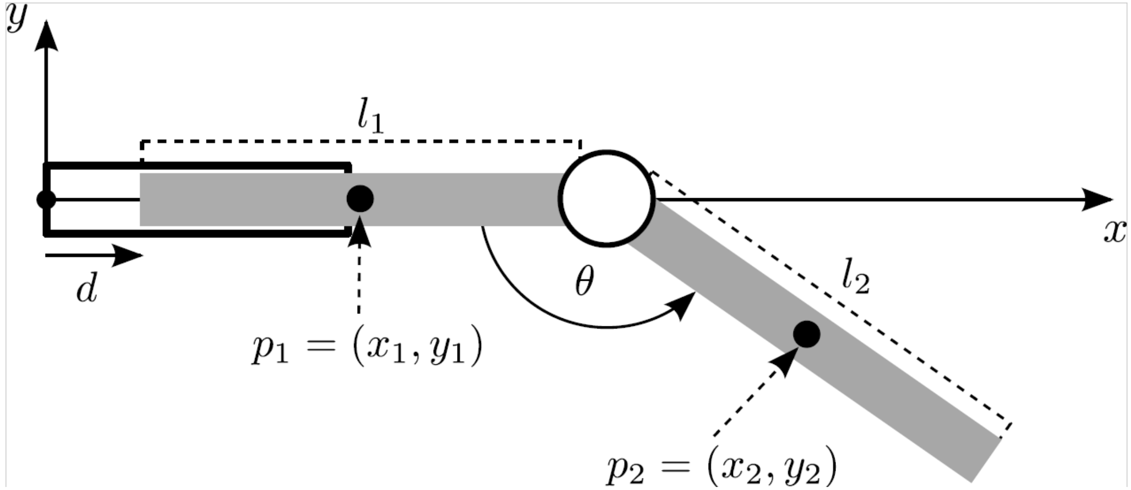
- Bewegungsgleichung:

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

- q_i: i-te Komponente der generalisierten Koordinaten
- τ_i: i-te Komponente der generalisierten Kräfte

Aufgabe 2.1: Kinetische Energie

$$E_{kin} = \frac{1}{2} m v^2$$



Kinetische Energien für s_1 und s_2:

$$E_{kin,1} = \frac{1}{2} m_1 \dot{d}^2$$

$$E_{kin,2} = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

Aufgabe 2.1: Kinetische Energie

$$p_2 = (x_2, y_2) = \left(l_1 + d + \frac{1}{2} l_2 \cdot \cos\theta, -\frac{1}{2} l_2 \cdot \sin\theta\right)$$

$$\dot{x}_2 = \frac{d}{dt}\left(l_1 + d + \frac{1}{2} l_2 \cdot \cos\theta\right)$$

$$= \dot{d} - \frac{1}{2} l_2 \dot{\theta} \sin\theta$$

$$\dot{y}_2 = \frac{d}{dt}\left(-\frac{1}{2} l_2 \sin\theta\right) = -\frac{1}{2} l_2 \dot{\theta} \cos\theta$$

Aufgabe 2.1: Kinetische Energie

$$E_{kin,2} = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) = \frac{1}{2} m_2 \left(\left(\dot{d} - \dot{\theta} \frac{1}{2} l_2 \sin\theta \right)^2 + \left(-\frac{1}{2} l_2 \dot{\theta} \cos\theta \right)^2 \right)$$

$$= \frac{1}{2} m_2 \left(\dot{d}^2 - \dot{d}\dot{\theta} l_2 \sin\theta + \frac{1}{4} l_2^2 \dot{\theta}^2 \sin^2\theta + \frac{1}{4} l_2^2 \dot{\theta}^2 \cos^2\theta \right)$$

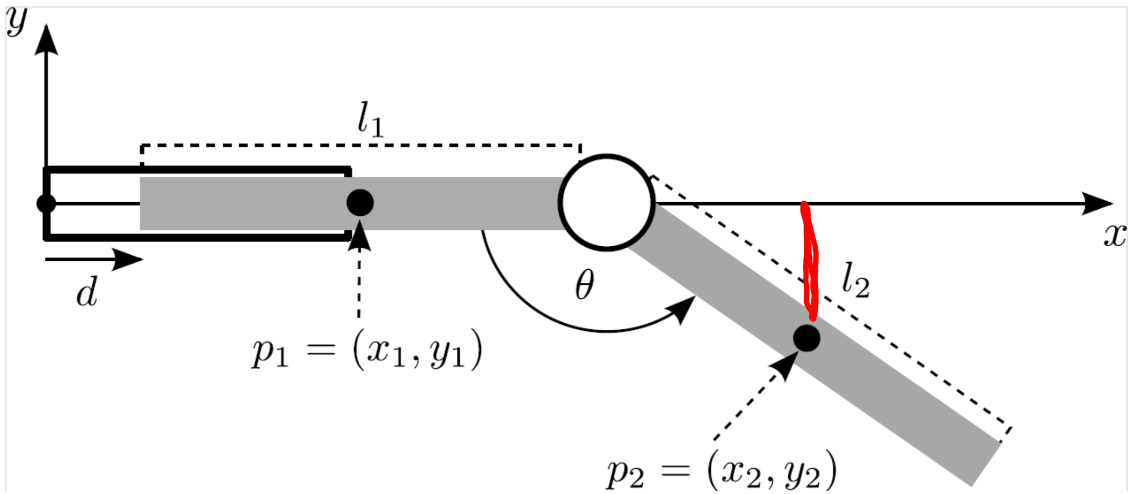
$$\sin^2\theta + \cos^2\theta = 1$$

$$= \frac{1}{2} m_2 \left(\dot{d}^2 - \dot{d}\dot{\theta} l_2 \sin\theta + \frac{1}{4} l_2^2 \dot{\theta}^2 \right)$$

$$= \frac{1}{2} m_2 \dot{d}^2 - \frac{1}{2} m_2 l_2 \dot{\theta} \dot{d} \sin\theta + \frac{1}{8} m_2 l_2^2 \dot{\theta}^2$$

Aufgabe 2.2: Potentielle Energie

$E_{pot} = mgh$



Potentielle Energien für s_1 und s_2:

$$E_{pot,1} = m_1 g y_1 = 0$$

$$E_{pot,2} = m_2 g y_2 = -\frac{1}{2} m_2 g l_2 \sin(\theta)$$

Aufgabe 2.3: Lagrange-Funktion

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$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = E_{kin}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - E_{pot}(\boldsymbol{q})$$

$$L = E_{kin,1} + E_{kin,2} - E_{pot,1} - E_{pot,2} =$$

$$= \frac{1}{2} m_1 \dot{d}^2 + \frac{1}{2} m_2 \dot{d}^2 - \frac{1}{2} m_2 l_2 \dot{d} \dot{\theta} \sin\theta + \frac{1}{8} m_2 l_2 \dot{\theta}^2 + \frac{1}{2} m_2 g l_2 \sin\theta$$

$$\frac{1}{2} m_1 \dot{d}^2 + \frac{1}{2} m_2 \dot{d}^2 = \frac{1}{2}(m_1 + m_2)\dot{d}^2$$

H2T

Aufgabe 2.3: Ableitungen der Lagrange-Funktion

$$L = \frac{1}{2}(m_1 + m_2)\dot{d}^2 - \frac{1}{2}m_2 l_2 \dot{d}\dot{\theta}\sin(\theta) + \frac{1}{8}m_2 l_2^2 \dot{\theta}^2 + \frac{1}{2}m_2 g l_2 \sin(\theta)$$

$$\frac{\partial L}{\partial \dot{d}} = (m_1 + m_2)\dot{d} - \frac{1}{2} m_2 l_2 \dot{\theta} \sin\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{d}} = (m_1 + m_2)\ddot{d} - \frac{1}{2} m_2 l_2 \left(\ddot{\theta}\sin\theta + \dot{\theta}^2 \cos\theta\right)$$

$$\frac{\partial L}{\partial d} = 0$$

Aufgabe 2.3: Ableitungen der Lagrange-Funktion

$$L = \frac{1}{2}(m_1 + m_2)\dot{d}^2 - \frac{1}{2}m_2 l_2 \dot{d}\dot{\theta}\sin(\theta) + \frac{1}{8}m_2 l_2^2 \dot{\theta}^2 + \frac{1}{2}m_2 g l_2 \sin(\theta)$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{4}m_2 l_2^2\,\dot{\theta} - \frac{1}{2}m_2 l_2 \dot{d}\sin(\theta)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{4}m_2 l_2^2 \ddot{\theta} - \frac{1}{2}m_2 l_2(\ddot{d}\sin(\theta) + \dot{d}\dot{\theta}\cos(\theta))$$

$$\frac{\partial L}{\partial \theta} = -\frac{1}{2}m_2 l_2 \dot{d}\dot{\theta}\cos(\theta) + \frac{1}{2}m_2 l_2 g cos(\theta)$$

Aufgabe 2.4: Bewegungsgleichung

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}}\right) - \frac{\partial L}{\partial d} =$$

$$= (m_1 + m_2)\ddot{d} - \frac{1}{2} m_2 l_2 \sin\theta \cdot \ddot{\theta} - \frac{1}{2} m_2 l_2 \dot{\theta} \cos\theta$$

Aufgabe 2.4: Bewegungsgleichung

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

$$\tau_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial d}$$

$$= \frac{1}{4} m_2 l_2^2 \ddot{\theta} - \frac{1}{2} m_2 l_2 \ddot{d} \sin(\theta) - \frac{1}{2} m_2 l_2 \dot{d}\dot{\theta} \cos(\theta)$$
$$-(-\frac{1}{2} m_2 l_2 \dot{d}\dot{\theta} \cos(\theta) + \frac{1}{2} m_2 l_2 g cos(\theta))$$

$$= \frac{1}{4} m_2 l_2^2 \ddot{\theta} - \frac{1}{2} m_2 l_2 \sin(\theta)\, \ddot{d} - \frac{1}{2} m_2 l_2 g cos(\theta)$$

Aufgabe 2.4: Bewegungsgleichung

$$\boldsymbol{\tau} = M(q)\ddot{\boldsymbol{q}} + \boldsymbol{c}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q})$$

$$\tau = \binom{\tau_1}{\tau_2} = \begin{pmatrix} (m_1 + m_2)\ddot{d} - \frac{1}{2}m_2 l_2(\ddot{\theta}\sin(\theta) + \dot{\theta}^2\cos(\theta)) \\ \frac{1}{4}m_2 l_2^2\ddot{\theta} - \frac{1}{2}m_2 l_2 \sin(\theta)\,\ddot{d} - \frac{1}{2}m_2 l_2 g cos(\theta) \end{pmatrix}$$

Aufgabe 2.4: Bewegungsgleichung

$$\boldsymbol{\tau} = M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q})$$

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} (m_1 + m_2)\ddot{d} - \frac{1}{2} m_2 l_2 (\ddot{\theta}\sin(\theta) + \dot{\theta}^2 \cos(\theta)) \\ \frac{1}{4} m_2 l_2^2 \ddot{\theta} - \frac{1}{2} m_2 l_2 \sin(\theta)\, \ddot{d} - \frac{1}{2} m_2 l_2 g cos(\theta) \end{pmatrix}$$

Aufgabe 2.4: Bewegungsgleichung

$$\boldsymbol{\tau} = M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q})$$

$q = \begin{bmatrix} d \\ \theta \end{bmatrix}$, $[d] = m$, $[\theta] = rad$

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} (m_1 + m_2)\ddot{d} - \frac{1}{2} m_2 l_2 (\ddot{\theta}\sin(\theta) + \dot{\theta}^2 \cos(\theta)) \\ \frac{1}{4} m_2 l_2^2 \ddot{\theta} - \frac{1}{2} m_2 l_2 \sin(\theta)\,\ddot{d} - \frac{1}{2} m_2 l_2 g \cos(\theta) \end{pmatrix}$$

$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$, $[\tau_1] = N$, $[\tau_2] = Nm$

Entspricht der allgemeinen Bewegungsgleichung:

$$\boldsymbol{\tau} = \underbrace{\begin{pmatrix} m_1 + m_2 & -\frac{1}{2} m_2 l_2 \sin(\theta) \\ -\frac{1}{2} m_2 l_2 \sin(\theta) & \frac{1}{4} m_2 l_2^2 \end{pmatrix}}_{M(q)} \underbrace{\begin{pmatrix} \ddot{d} \\ \ddot{\theta} \end{pmatrix}}_{\ddot{q}} + \underbrace{\begin{pmatrix} \frac{1}{2} m_2 l_2 \dot{\theta}^2 \cos(\theta) \\ 0 \end{pmatrix}}_{c(\dot{q}, q)} + \underbrace{\begin{pmatrix} 0 \\ -\frac{1}{2} m_2 l_2 g \cos(\theta) \end{pmatrix}}_{g(q)}$$

Matlab für die nächsten Übungen

- Installationsanleitung für Studierende am KIT
  - https://www.scc.kit.edu/produkte/3841.php
- Robotics Toolbox (Peter Corke)
  - http://petercorke.com/wordpress/toolboxes/robotics-toolbox#Downloading_the_Toolbox
  - Die .mltbx Datei herunterladen
  - Aus dem Matlab-Filexplorer öffnen
- Selber ausprobieren: Übungsblatt 1 in Matlab lösen